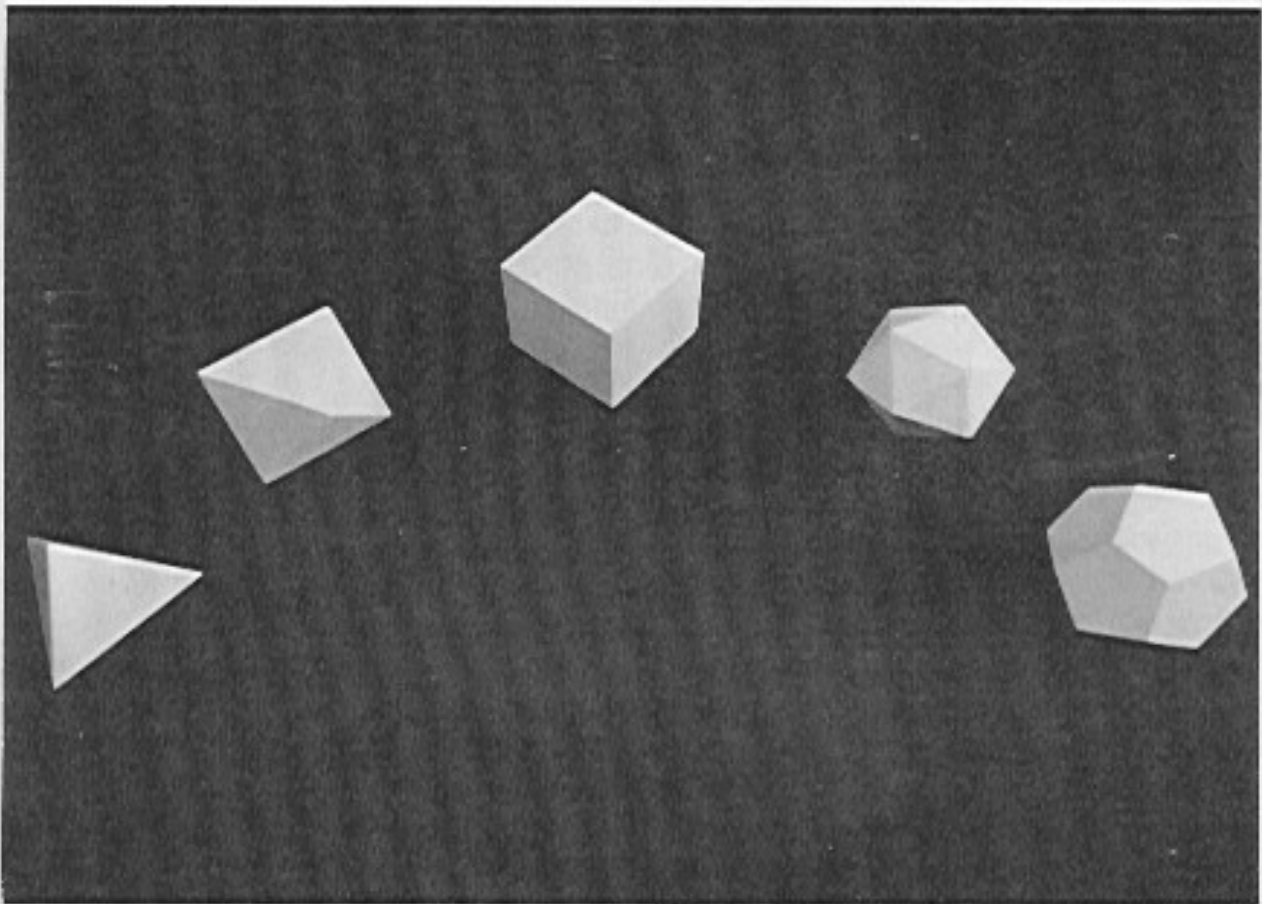


The Five Regular Platonic Bodies



The five 'regular' Platonic Bodies to be found in some Chapters. The illustration depicts them in their correct sequence and, reading from left to right, may be explained:

4	Tetrahedron	=	1 Triple Tau	—	symbolises FIRE] The Four Elements
8	Octahedron	=	2 Triple Taus	—	symbolises AIR	
6	Hexahedron	=	3 Triple Taus	—	symbolises EARTH	
20	Icosahedron	=	5 Triple Taus	—	symbolises WATER	
12	Dodecahedron	=	9 Triple Taus	—	symbol of the Universe	

Prepared for members of The Hauraki Royal Arch District
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The Five Regular Platonic Bodies

Companion Haggai, during the Symbolical Lecture which he delivers subsequent to the Ceremony of Exaltation, alludes to the "Five Regular Platonic Bodies", which he explains as "representing the Four Elements and the Sphere of the Universe".

The "Five Regular Platonic Bodies" are the Dodecahedron (a solid having twelve faces), representing the Sphere of the Universe; the Cube (a solid having six equal square faces), representing Earth; the Tetrahedron (a solid bounded by four triangles), representing Fire; the Octahedron (a solid figure contained by eight equal equilateral triangles), representing Air; and the Icosahedron (a solid figure bounded by twenty plane faces), representing Water.

These Regular Platonic Bodies may properly be regarded as being included among the Symbols and Ornaments of a Royal Arch Chapter, and they are always to be found ranged in their proper order on the floorcloth at the Aldersgate Chapter of Improvement.

Unfortunately, however, they are sometimes conspicuous by their absence in the majority of Regular Chapters, and as a result Companion Haggai's reference thereto probably leaves the newly-exalted novice, for whose special benefit the Lecture is being delivered, in a state of perplexing bewilderment, unless he happens to be an accomplished geometrician, or unless the geometrical studies of his school-days left lasting impressions in his mind.

To cover the subject comprehensively within the space of a few pages is not easy, and it will, perhaps, be most convenient to deal with it under three headings:

- (1) the nature and construction of the Bodies;
- (2) the Origin of the Four Elements according to the Platonic Theory;
- (3) how the Elements are considered to be represented by the regular solids.

The nature and construction of the Bodies

The so-called Platonic Bodies are the five geometric solids, and are, in fact, the only strictly regular solids which can be constructed. They were discovered (or first described) by Pythagoras about the year 40 BC.

Pythagoras held that there were two elementary triangles which were to be considered the most perfect, viz, the right angled triangle having the shorter side containing the right angle one half of the hypotenuse, and the isosceles right angled triangle which has the sides containing the right angle equal.

From these two elementary triangles four of the regular bodies can be constructed. Two of the first type are placed with their long sides together, and then three of these pairs are assembled to form an equilateral triangle.

When it is said that the so-called Platonic Bodies are the only regular solids which can be constructed, it must be borne in mind that the word "regular" is used in the sense that Euclid employed it.

Euclid's definition of a regular plane figure is one which has all its sides equal; and a regular solid is one which is contained by equal and similar plane surfaces, so that it has all its surfaces equal and equilateral, and all its solid angles equal.

The simplest of all plane figures is a triangle, and it is interesting to ascertain what solids can be constructed with equilateral triangles.

The smallest solid angle is one contained by three plane angles and it is found that one can be formed with three of these angles and the base is another equilateral triangle. This forms the *Tetrahedron*, which is contained by four equal equilateral triangles.

Again it is found that four of these equilateral triangles will form a solid angle, the base of which is a square, and it is at once evident that if another four triangles are put together and applied to these a solid is formed - the *Octahedron*.

Again, five equilateral triangles will form a solid angle, the base of which is a regular pentagon and

the alternative points being joined, produces the pentagon. If another triangle be joined to each side of this pentagon, and the whole be repeated, it will be found that the two will fit together and form a solid, contained by twenty equilateral triangles the - *Icosahedron*.

If six equilateral triangles are placed together they will all lie in a plane, because six times 60 degrees equals 360 degrees, and therefore no solid angle can be formed.

The next regular plane figure to be constructed is the square, which is derived from the second of the elementary triangles by assembling four of them with their right angles together in the centre. Three of these squares will form a solid angle, which leads at once to the *Cube*, which is bounded by six equal squares. Four squares together will lie in a plane, so that no other solid can be formed with equal square faces.

Taking the regular five-sided figure, the pentagons it is found that the solid angle can be formed with three pentagons, which leads to the fifth solid - the *Dodecahedron* - which is contained by twelve equal and equilateral pentagons. As four pentagons together amount to more than 360 degrees no other solid is possible.

Taking the next plane figure, that with six sides, the regular hexagon, it is seen that three together lie in a plane, and no solid angle can be formed; consequently no solid. Any regular figure with more than six sides has an angle so large that three of them would exceed 360 degrees, and construction of a solid angle is therefore, impossible.

Other solids have been constructed which are, in some respects, regular, in that they are symmetrical, but in the strict sense of the word "regular", the five mentioned are the only ones possible.

The Origin of the four elements

We now come to the second of our three headings, the Origin of the four elements, viz, Fire, Air, Water and Earth.

In one of the Platonic Dialogues, known as the "Timaeus", is to be found a description of the formation of the Universe by the Deity. According to the Platonic Theory everything that is created, or, as the Platonists preferred, everything that is generated, must be visible or tangible, and must be composed of visible or tangible elements by which they meant that it must be capable of appreciation by the senses.

Timaeus argues thus:

"Whatever is created must be visible or tangible, but nothing can be visible without the aid of Fire. Nothing can be tangible unless solid, and nothing is more solid than Earth."

Fire and Earth were, therefore, regarded as the two principal Elements.

The Platonists held that no two things can cohere without the aid of a third medium, and that one such medium was necessary to form a superficies, but the two media were required to form a solid. They therefore introduced the two Elements, Air and Water, between Fire and Earth.

The Platonist theory was that, in order that the Universe might be as perfect as possible, the Deity combined these four Elements in such a manner that there might be an exact analogy between the four, i.e. as Fire is to Air, so is Air to Water, and as Air is to Water, so is Water to Earth, thus forming a regular and harmonious gradation from the lightest and most penetrating of the Elements to the heaviest and most obtuse.

The Platonists reasoned that the Deity thus formed the Universe from the Four Elements, and, as the Sphere was considered the most perfect form since it contains within itself all other forms, He made the Universe in the form of a Sphere.

How the elements are represented by the solids

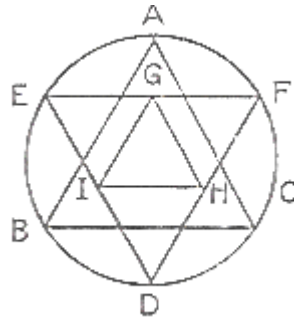
Our third heading is as to how the Four Elements may be considered to be represented by the four regular solids.

The Platonists held that the Elements were tangible bodies, and that all bodies possess some degree of solidity. They therefore likened the Elements to the most perfect of the solid bodies,

which are the regular ones, their theory being that all regular and symmetrical forms must be more perfect than irregular ones.

Of the five regular solids the *Cube* is the most stable and immovable; therefore it was selected to represent Earth. The *Icosahedron*, being the heaviest, was assigned to Water. The *Tetrahedron*, the sharpest and most agile, was chosen to represent Fire. The *Octahedron*, the least stable, was selected for Air. The *Dodecahedron*, which approached nearest to the Sphere, was chosen to represent the Universe.

In the course of his Lecture, Companion Haggai explains to the newly exalted Companion that the Jewel of the Order forms by its intersections a given number of angles which, when reduced to their amount in right angles, will be found "equal to the Five Regular Platonic Bodies". Previously he has explained how a Triangle, divided into four smaller ones, will contain in all eight right angles, equal to the mysterious triple tau. The accompanying diagram of the Jewel is necessary to explain what follows.



The central triangle GHI resolved into its elements in the manner described by Companion Haggai in the Lecture, will contain eight right angles, equal to one triple tau, and is therefore equal to the *Tetrahedron*, which is contained by four equilateral triangles.

Taking the two larger triangles ABC, DEF, these, resolved as before, will contain sixteen right angles, or two triple tau which equals the *Octahedron*, which is contained by eight equilateral triangles.

Taking next the two large triangles ABC, DEF, and the small central one GHI, these, resolved as before, will contain twenty-four right angles, or three triple tau, which are equal to the *Cube*, which is contained by six squares.

Considering next the large inverted triangle DEF, as divided into four smaller triangles, adding to the other large triangle, and resolving all five as before, they will now contain forty right angles, or five triple tau, equalling the *Icosahedron*, which is contained by twenty equilateral triangles.

Finally, if we take the six small triangles round the circle, and the central one, then, resolving them all as before, we find they contain fifty-six right angles. Add to these the external angles of the six small triangles, considering each to have two external angles (twelve in all), each of which is two-thirds of two right angles, and we get sixteen right angles. This yields a total of seventy-two right angles, or nine triple tau, equalling the *Dodecahedron*, which is contained by twelve equal pentagons, each pentagon containing six right angles.

Companions,

While it is NOT usual to see the Five Regular Platonic Bodies on display in a Royal Arch Chapter holding under the New Zealand, yet the second booklet of our official ritual refers to them in the *Address of the Second Chair*. In addition, those of our members who visit the English Constitution Chapters, may have noticed the Bodies set out on the pavement (floor of the vault) and wondered what they were and what their symbolism is.

This booklet is intend to remedy this and allow us all to make another advancement ...

Fraternally,

Jim Anderson